Section A [45 *marks*]

Answer **all** *questions.*

1 The function f is defined by

$$f(x) = \begin{cases} \dfrac{64 - x^3}{x - 4}, & x < 4, \\ (1 - m^2)x^2, & x = 4, \\ n\sqrt{x - 1}, & x > 4. \end{cases}$$

Determine the exact value of the constants m and n such that the function f is continuous at $x = 4$. [7]

2 A curve is defined parametrically by $x = \dfrac{9t^2 - 1}{3t}$ and $y = \dfrac{9t^2 + 9t + 1}{3t}$, where $t \neq 0$.

(*a*) Find the coordinates of the points where the tangent line to the curve is parallel to x-axis. [7]

(*b*) Is there any tangent line to the curve which is parallel to y-axis? Justify your answer. [2]

3 Find the exact value of $\displaystyle\int_0^{\ln\frac{\pi}{4}} e^x \cot(2e^x)\,dx$. [6]

4 Find the particular solution of the differential equation $x^2 e^{x^4} \dfrac{dy}{dx} + 5xye^{x^4} = 1$, for $x > 0$, with the condition $y = 0$ when $x = 1$. [8]

5 Using Maclaurin series for e^x and $\cos x$, find Maclaurin series for $e^x(1 + \cos 2x)$ up to the terms in x^4. [3]

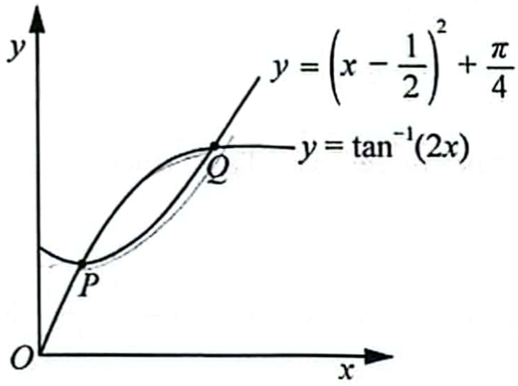
Hence, evaluate $\displaystyle\lim_{x \to 0} \frac{e^x \cos^2 x - 1}{x}$. [4]

6 Use differentiation to show that the iteration $x_{n+1} = (2.1 - 4e^{-2x_n})^2 - 1$ converges to the root of the equation $4e^{-2x} + \sqrt{x + 1} = 2.1$ in the interval [3, 4]. [3]

Hence, find the root using initial approximation $x_0 = 4.0$ correct to three decimal places. [5]

You may answer all the questions, **but** *only the first answer will be marked.*

7 Two curves with the equations $y = \tan^{-1}(2x)$ and $y = \left(x - \frac{1}{2}\right)^2 + \frac{\pi}{4}$ in the first quadrant are shown in the graph below.



Both curves intersect at two points, P and Q (1.099, 1.144), where P is the minimum point of the curve $y = \left(x - \frac{1}{2}\right)^2 + \frac{\pi}{4}$.

(a) State the coordinates of P. [1]

(b) Calculate the area of the region bounded by the curves $y = \tan^{-1}(2x)$ and $y = \left(x - \frac{1}{2}\right)^2 + \frac{\pi}{4}$. [9]

(c) Calculate the exact volume generated when the region bounded by the curve $y = \tan^{-1}(2x)$, $y = \frac{\pi}{4}$ and the origin O is revolved completely about the y-axis. [5]

8 Assume that the rate of elimination of caffeine from the body is k times the mass of caffeine, x mg, of the remaining active amount of caffeine at time t hours, where k is a constant. Once an average-sized cup of coffee is consumed completely, the initial amount of caffeine in the body is x_0 mg.

(a) (i) State a differential equation which describes the above situation. Hence, solve the differential equation. [4]

(ii) If the half-life of caffeine in the body is 5 hours, determine the value of k. [3]

(iii) Sketch the graph of x against t. [2]

(b) Assume that an average-sized cup of coffee contains 95 mg of caffeine and the coffee is consumed 8 hours ago,

(i) how much caffeine remains in the body? [2]

(ii) determine the time taken to eliminate 98% of the caffeine from the body. Can the amount of caffeine totally be eliminated from the body? Justify your answer. [4]